A Hybrid BRNN-ARIMA Model for Financial Time Series Forecasting

Muhammad Najamuddin¹ and Samreen Fatima²

Abstract:

The accurate forecasting of time series is challenging and for exchange rate is more problematic as well. Because it is difficult to predict as they continuously fluctuate during trading hours. Exchange rate forecasting is a vital financial problem in the recent era. It is extensively acknowledged that exchange rate stability implies macroeconomic stability. In this study, a hybrid model is proposed to forecast exchange rates. Bayesian regularized neural network (BRNN) model is assembled with Autoregressive integrated moving average model (ARIMA) and develop hybrid BRNN-ARIMA model. Furthermore, the comparison of the proposed hybrid model has been done with standalone BRNN, standalone ARIMA, and random walk model. Quarterly exchange rate data from 1970Q1 to 2021Q2 of six countries comprising developed (UK, Canada, and Singapore) and developing (Pakistan, India, and Malaysia) are forecast. To evaluate the performance of these models RMSE, MAE and MAPE are applied. The results indicate that the proposed hybrid BRNN-ARIMA model outperforms the other studied model in forecasting exchange rates.

Keywords: Exchange rate, Bayesian regularized neural network, ARIMA, random walk.

1. Introduction

Exchange rate prediction has been an important problem in the recent era. The stability of the exchange rate implies macroeconomic stability. It is considered an important measure of the economy. Financial systems are highly influenced by exchange rate fluctuations. Therefore, providing an accurate prediction of exchange is difficult as it is varying continuously during trading hours (Brooks, 1997; Diabold, Gardeazabal, and Yilmaz, 1994; Trapletti, Geyer, and Leisch 2002; Kilian and Taylor, 2003). Since the pioneering work of forecasting exchange rates by the random walk model (Meese and Rogoff, 1983), gave rise to extensive research to provide accurate exchange rate forecasting. Furthermore, research has been done to explore the fundamentals and trends of exchange rates using time series models such as ARIMA, GARCH, TAR, and SETAR models (see Hsieh, 1988; Engel and Hamilton, 1990; Cheung and Wong, 1997; Brooks, 1997; Kilian and Taylor, 2003; Gharleghi, Shaari, and Sarmidi, 2014; Khashei and Sharif, 2020). Moreover, machine learning models for instance ANN, SVM, genetic algorithms, etc. to predict exchange rates are used (see Albers et. al., 1996; Kastra and Boyd, 1996; Yao and Tan, 2000; Vojinovic and Keeman, 2001; Kamruzzaman and Sarker, 2003; Plakandaras, Papadimitriou, and Gogas, 2015; Sun, Wang and Wei, 2020). Nowadays, it is recommended theoretically and empirically that the combination of different methods can provide an effective and efficient forecast (see

¹Department of Statistics, Federal Urdu University of Arts, Sciences & Technology, Karachi, Pakistan
²Department of Statistics, University of Karachi, Pakistan
Corresponding Author: muhammad.najam@hotmail.com

This paper focuses on forecasting daily exchange rates using a hybrid BRNN-ARIMA model with a standalone BRNN model, standalone ARIMA model, and RW model for six countries comprising developed (UK, Canada, and Singapore) and developing (Pakistan, India, and Malaysia). Mainly this paper investigates the forecasting performance of the hybrid model, machine learning model, and time series models in forecasting exchange rates. Also, we have made the comparison of these models for developed and developing countries separately. The reason for carrying out such analysis is that the exchange rate forecasts can provide useful information for fund managers in their decision-making process, borrowers, corporate treasurers, specialized traders, and the government.

2. Literature Review

McCrae et. al. (2002) evaluated the forecasting performance of the co-integration-based ECM model with the ARIMA model. They forecasted five Asian exchange rates using daily data from January 1985 to February 1997. One to 40 days out-of-sample forecasting horizons were used. They found that for short-term forecasting, the ARIMA model gives a better forecast. While for medium-run forecasting Johansen’s co-integration-based ECM model gives a better forecast. Rapach and Wohar (2002) investigated the long-run phenomena in forecasting exchange rates. A comparison of Johansen’s vector error correction model has been made with a random walk model for fourteen selected industrialized countries. The results are in favor of the vector error correction model. Kashif et. al. (2008) forecasted Pakistani exchange rates with ARIMA, GARCH, and state space models. The daily data of Pakistani exchange rates against the US Dollar is considered. They found that the exchange rate model is better predicted with state space models. Rasheed, Asad, and Imam (2020) examined the exchange rates of the Pakistani rupee against the US dollar. They compared univariate and multivariate time series models. They forecasted exchange rates with exponential smoothing, Naive, ARIMA, and ARDL Co-integration models. They found that the exponential smoothing model better predicts the exchange rate.

The forecasting comparison of neural network models has been done with RW and autoregressive models by Panda and Narasimhan (2007). Weekly Indian exchange rate per US dollar data is used for forecasting. Empirical results showed that the neural network model gives superior performance. Another comparison of feed-forward and the recurrent neural network has been made with the ARIMA model in forecasting exchange rates by Kuan and Lieu (1995). In which they used predictive stochastic complexity for network selection of neural networks and found better out-of-sample forecasting of the proposed technique. Some other examples of exchange rates forecasting by ANN are Tang and Fishwick, 1993; Jhee and Lee, 1993; Hill et. al., 1996; Wang and Leu, 1996; Kamruzzaman and Sarker, 2003; Ni and Yin, 2009; Qian and Rasheed, 2010; Shen et. al., 2015.

Nowadays, hybrid approaches are more in use to get a better forecast. In hybrid approaches, two or more linear and nonlinear approaches are used. Wang et. al. (2013) proposed a hybrid model consisting of ARIMA and ANN. In the proposed model, they used two hybrid models: the additive model and the multiplicative model. In the additive model, they were adding the linear and nonlinear components of the forecast. While in the multiplicative model, they were multiplying the linear and nonlinear components of the forecast. They found that the multiplicative model gives superior performance over the additive hybrid model, ANN, and ARIMA models. Khashei and Sharif (2020) combined ARIMA and Kalman filter-based ANN to
forecast exchange rates. The results of the proposed model are compared with AR, ANN, and RW models and found the superiority of the proposed model. Ince and Trafalis (2006) used the combination of parametric (ARIMA and VAR) and nonparametric models (SVR and ANN models) to forecast exchange rates. They showed that the SVR model gives a better forecast in both cases, where input was taken from ARIMA and VAR models. Mucaj and Sinaj (2017) proposed a hybrid ARIMA-ANN model, in which the ANN model is applied to the forecast generated from the ARIMA model. They compared the proposed model with ARIMA and the nonlinear autoregressive neural network model and found that the out-of-sample forecasting performance of the proposed model is better. Chen and Lin (2007) introduced a hybrid model consisting of a fuzzy membership function and neural network model in exchange rate prediction. Liao et. al. (2020) used a neural network model with their proposed hybrid model consisting of Markov switching and an EGARCH model called the NN-MS Beta t-EGARCH model. The proposed model was compared with various traditional models and resulted in superior forecasting performance of exchange rates. Similarly, Khashei, Hejazi, and Bijari (2008) suggested the combination of ANN and fuzzy regression to get better results. Areekul (2010) and Saeed Matroushi (2011) combined ARIMA and the multilayer perceptron model to get a better alternative. Aladage (2009) used a combination of Eivan’s RNN and ARIMA models to produce the hybrid model in getting a better forecast of exchange rates.

The reviewed literature studies showed the capability of exchange rate forecasting with a random walk, Johansen vector error correction model, ARIMA, VAR, ARFIMA, ANN, and GARCH models. Also, exchange rate forecasting has been made by hybrid models in which a combination of two or more linear and nonlinear models are used for example ARIMA with ANN, Fuzzy inference system and ANN, etc. While forecast precision of the hybrid model based on Bayesian regularized neural network and ARIMA models have not been investigated. Furthermore, many studies that used real data in exchange rate forecasting came from developed countries. The main two contributions of this paper in the literature can be described as. First, forecasting comparison of hybrid model based on Bayesian regularized neural network and ARIMA model namely hybrid BRNN-ARIMA model along with machine learning technique; BRNN model, and time series models; ARIMA and random walk models, which has never done before. Second, most of the studies used data from developed countries i.e., the G-7 countries, the UK, and the USA to forecast macroeconomic variables. This paper used data from six countries including both developed and developing countries namely the UK, Canada, Singapore, Pakistan, India, and Malaysia.

3. Forecasting Techniques

3.1 Bayesian Regularized Neural Network Model

Bayesian regularization to ANN is used to overcome the overfitting issue in ANN. A typical ANN model is shown in Fig. 1. ANN and Bayesian methods are linearly related for the determination of optimal regularization parameters automatically. In 1992, a Gauss-Newton approximation method for the estimation of regularization parameters of the posterior distribution was suggested by Mackay (1992). Maximization of the objective function with parameters α and β has been done in this method. Levenberg–Marquardt algorithm is applied and an iterative solution for α and β as proposed by Mackay (1992) has been used in this paper. The model is given by:

\[ y_i = g(x_i) + e_i = \sum_{k=1}^{s} w_k g_k(b_k + \sum_{j=1}^{p} x_{ij} \beta_j^{[k]} + e_i, i = 1,2,\ldots,n \]  

then it minimizes

\[ F = \beta E_D + \alpha E_W \]  

Where,

\[ e_i \sim N(0, \sigma_e^2) \]

\[ \beta_j^{[k]} \] is the weight of the \( j \)th input to the net, \( j = 1,2,\ldots,p \). \( g_k(\cdot) \) is the activation function, in this implementation
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\[ g_k(x) = \frac{e^{2x} - 1}{e^{2x} + 1}, \quad E_D = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \]
i.e., the sum of squares of errors
\[ E_W \] is the network parameters sum of squares (weights and biases).

\[ \beta = \frac{1}{2\sigma^2} \quad \text{and} \quad \alpha = \frac{1}{2\sigma^\theta}, \sigma^\theta \] is a dispersion parameter for weights and biases.

3.2 Hybrid Model

To deal with the linear and nonlinear pattern of exchange rates, a combination of BRNN and ARIMA models is proposed in this paper. The linear component is captured by the ARIMA model. The residual of the ARIMA model is
\[ e_t = y_t - \hat{Y}_{lt} \]
Where \( \hat{Y}_{lt} \) represents the forecasted value at time \( t \) from the ARIMA model. Then the residuals from the ARIMA model are used to model the BRNN model which will capture the nonlinear component. Forecasts from the BRNN model, denoted by \( \hat{Y}_{nt} \), are then added to the predicted values from the ARIMA model to get the combined forecast shown in Fig. 2.

3.3 Random Walk Model

The random walk model is the vital and popular statistical model for empirical finance. It has been widely used to forecast exchange rates since the pioneering work has been done by Meese and Rogoff (1983). In which they showed that the random walk model is best fitted in exchange rate prediction. In this model, the best forecast value is assumed to be the most recent observation. A simple RW model can be stated mathematically as
\[ ER_t = ER_{t-1} + \varepsilon_t \quad (3) \]
Where, \( ER_t \) is the exchange rate observed at the beginning of time \( t \) and \( \varepsilon_t \) is the noise term. The distribution of noise term is identically and independently distributed normal variable having mean zero and a constant variance \( \sigma^2 \).

3.4 Autoregressive Integrated Moving Average (ARIMA) Model

ARIMA model, produced by Box et al. (1994), is one of the popular and broad usage models for time series modeling and forecasting. It has been applied to model and forecasting many economic and financial time series for example exchange rates. The ARIMA model forecast the time series by using available information in past and current values of time series. The ARIMA (f, d, g) model contains ‘f’ autoregression parameters, ‘g’ moving average parameters, and ‘d’ is the differencing order to make the stationary time series. For ARIMA modeling, time series must be stationary. The order of components of the ARIMA model is selected based on ACF, PACF, AIC, and BIC criteria. To represent the ARIMA model we consider the exchange rate \( ER_t \).
\[ \phi(L)(1-L)^d ER_t = \mu + \theta(L)u_t \quad (4) \]
Where \( \mu \) is the intercept term,
\[ \phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3 - \cdots - \phi_f L^f \]
\[ \theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \theta_3 L^3 - \cdots - \theta_g L^g \]
\( \phi_i \)'s is the autoregression coefficients and \( \theta_i \)'s is the moving average coefficients and \( u_t \) are errors, which are identically distributed with zero mean and constant variance.
4. The Data

Quarterly data is considered from the first quarter (Q1) of 1970 to the second quarter of 2021 (Q2) which includes 206 observations. For model specification and estimation, 75% of data is employed from 1970Q1 to 2008Q3 which consists of 155 observations, and the forecast evaluation is conducted for the remaining 25% of data from 2009Q4 to 2021Q2 having 51 observations.

The quarterly data (1970Q1-2021Q2) of both developed and developing countries are employed including the following countries. UK, Canada, and Singapore for developed countries while Pakistan, India, and Malaysia for developing countries. All the exchange rates currency data are considered as local currency per US dollar. The source of the data is International Financial Statistics (IFS) (https://data.imf.org/).

Forecast accuracy is evaluated by Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). The formulas are given by:

\[ RMSE = \sqrt{\frac{1}{H} \sum_{t=1}^{H} (Y_t - \hat{Y}_t)^2} \]  

\[ MAE = \frac{1}{H} \sum_{t=1}^{H} |Y_t - \hat{Y}_t| \]  

\[ MAPE = \frac{100}{H} \sum_{t=1}^{H} \frac{|Y_t - \hat{Y}_t|}{Y_t} \]  

Where forecast horizon is represented by \( H \), actual and forecast values are represented by \( Y_t \) and \( \hat{Y}_t \) respectively.

5. Results and Discussion

For analysis from the BRNN model, the number of neurons and hidden layers are selected on which errors are minimized. We took a range from 1 to 10 for the number of neurons and 1 to 6 for hidden layers. The BRNN model is applied for all possible combinations of the number of neurons and hidden layers and then that model is selected which shows minimum error, as shown in Table 1. And that selected model is used for forecasting. While the selection of the order of the ARIMA model is based on the smallest AIC and BIC criteria. Table 1 shows the orders of the ARIMA model.

In Table 2 and Fig. 3, the forecasting results of the hybrid BRNN-ARIMA, BRNN, ARIMA, and RW models are given for developed countries. RMSE, MAE, and MAPE are calculated for all techniques to forecast the exchange rates of all countries.
In a comparison of the BRNN model and ARIMA model, forecast errors associated with the BRNN model are lower than the ARIMA model for the UK, and the forecast ability of the BRNN model is improved by 85% and 99% than ARIMA and random walk model on the average. While forecast errors associated with the ARIMA model are lower than the BRNN model for Canada and Singapore. For Canada forecast ability of the ARIMA model is improved by 82% and 94% for BRNN and random walk model. And for Singapore, the

<table>
<thead>
<tr>
<th>Countries</th>
<th>Number of neurons</th>
<th>Number of Hidden Layers</th>
<th>ARIMA order</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>3</td>
<td>4</td>
<td>(3, 1, 2)</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>3</td>
<td>(4, 1, 1)</td>
</tr>
<tr>
<td>Singapore</td>
<td>4</td>
<td>4</td>
<td>(0, 1, 0)</td>
</tr>
<tr>
<td>Pakistan</td>
<td>4</td>
<td>8</td>
<td>(1, 1, 0)</td>
</tr>
<tr>
<td>India</td>
<td>4</td>
<td>6</td>
<td>(4, 1, 4)</td>
</tr>
<tr>
<td>Malaysia</td>
<td>3</td>
<td>5</td>
<td>(3, 1, 2)</td>
</tr>
</tbody>
</table>

Table 1: Selected number of neurons and hidden layers for BRNN model

<table>
<thead>
<tr>
<th>DEVELOPED COUNTRIES</th>
<th>Forecasting techniques</th>
<th>Hybrid BRNN-ARIMA</th>
<th>BRNN</th>
<th>RW</th>
<th>ARIMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>RMSE</td>
<td>0.003426</td>
<td>0.005947</td>
<td>0.921908</td>
<td>0.036033</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.00191</td>
<td>0.003836</td>
<td>0.736575</td>
<td>0.026027</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.292257</td>
<td>0.501225</td>
<td>108.8179</td>
<td>3.820909</td>
</tr>
<tr>
<td>Canada</td>
<td>RMSE</td>
<td>0.005105</td>
<td>0.2629</td>
<td>0.981329</td>
<td>0.054536</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.001906</td>
<td>0.2589</td>
<td>0.794888</td>
<td>0.043278</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.157671</td>
<td>22.622</td>
<td>66.95992</td>
<td>3.642427</td>
</tr>
<tr>
<td>Singapore</td>
<td>RMSE</td>
<td>0.000905</td>
<td>0.034729</td>
<td>1.037512</td>
<td>0.039323</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.000567</td>
<td>0.03372</td>
<td>0.836052</td>
<td>0.029985</td>
</tr>
<tr>
<td></td>
<td>MAPE</td>
<td>0.041453</td>
<td>2.5502</td>
<td>62.63986</td>
<td>2.207266</td>
</tr>
</tbody>
</table>

Table 2: RMSE, MAE, and MAPE of the forecast of exchange rate for developed countries

Fig. 3: Graphical display of RMSE, MAE, and MAPE of four forecasting methods for developed countries
forecast ability of the ARIMA model is improved by 4% and 96% respectively than BRNN and random walk model. Forecast errors of all evaluation criteria associated with the random walk model are higher than all other techniques. In the comparison of the hybrid BRNN-ARIMA model with other models, the forecast ability of the hybrid BRNN-ARIMA model over BRNN, ARIMA, and RW models is improved on average by 46%, 92%, and 99% respectively for the UK, while the forecast ability for Canada associated with hybrid BRNN-ARIMA model than BRNN, ARIMA, and random walk model is improved by 99%, 93%, and 99% respectively on the average. And for Singapore, the forecast ability of the hybrid BRNN-ARIMA model than BRNN, ARIMA, and random walk model is improved by 98%, 98%, and 99% respectively on average. The results showed that the hybrid BRNN-ARIMA model outperformed all the other forecasting methods for all developed countries.

In Table 3 and Fig. 4, the forecasting results of the hybrid BRNN-ARIMA, BRNN, ARIMA, and RW models are given for developing countries. RMSE, MAE, and MAPE are calculated for all techniques to forecast the exchange rates of all countries. In the comparison of the BRNN and RW model, forecast errors associated with the RW model are lower than the BRNN model for Pakistan and India, forecast ability of the random walk model than BRNN and ARIMA model is improved by 30%, and 16% respectively on the average for Pakistan and 55% and 3% respectively on the average for India. While forecast errors associated with the RW model are lower than the BRNN model for Malaysia and the forecast ability of the BRNN model than random walk and ARIMA model is improved on average by 97% and 78% respectively. Forecast errors of all evaluation criteria associated with the ARIMA model are higher than all other techniques. In all cases, the ARIMA model is giving tough competition to all other models for all countries. The forecast errors associated with ARIMA models are very close to the random walk model for Pakistan and India. In the comparison of the hybrid BRNN-ARIMA model with other models, the forecast ability of the hybrid BRNN-ARIMA model over BRNN, ARIMA, and RW models is improved on average by 45%, 37%, and 24% respectively for Pakistan, 96%, 94%, and 97% respectively for India and 86%, 97% and 99% respectively for Malaysia. The results showed that the hybrid BRNN-ARIMA model outperformed all the other forecasting methods for all developing countries as well.

On comparing forecast errors of all forecasting techniques from developed and developing countries, forecast errors of all techniques from developed countries are lower than the forecast errors from developing countries except for Malaysia. From this, it can be concluded that forecasting exchange rates for developed countries are more accurate than for developing countries. This may be because Pakistan and India are suffering from unstable political situations and the exchange rates of these countries are much more diverse than in other countries since economic conditions in the UK, Canada, Singapore, and Malaysia are stable, which directly impacts the stability of exchange rates.

6. Conclusions
It is extensively acknowledged that macroeconomic stability is influenced by exchange rate stability which impacts economic growth positively. Economic efficiency can be reduced due to uneven exchange rates. So accurate forecasting of exchange rates is important for the development of the country. In this paper, exchange rates forecasting is made for three developed and three developing countries using a hybrid BRNN-ARIMA model, Bayesian regularized neural network model, autoregressive integrated moving average model, and random walk model. Overall results are in favor of the hybrid BRNN-ARIMA model. Other than the hybrid BRNN-ARIMA model results are mixed. Random
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walk model is beaten by other models for developed countries, while for developing countries especially for Pakistan and India random walk model gives a superior performance as compared to other models. Overall, the Bayesian regularized neural network model gives a better forecast for stable economies. In general, the proposed hybrid BRNN-ARIMA model outperforms all other techniques in forecasting exchange rates for both developed and developing countries.

Table 3: RMSE, MAE and MAPE of forecast of exchange rate for developing countries

<table>
<thead>
<tr>
<th>DEVELOPING COUNTRIES</th>
<th>Forecasting techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hybrid BRNN-ARIMA</td>
</tr>
<tr>
<td>Pakistan</td>
<td><strong>RMSE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAPE</strong></td>
</tr>
<tr>
<td>India</td>
<td><strong>RMSE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAPE</strong></td>
</tr>
<tr>
<td>Malaysia</td>
<td><strong>RMSE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAE</strong></td>
</tr>
<tr>
<td></td>
<td><strong>MAPE</strong></td>
</tr>
</tbody>
</table>

Fig. 4: Graphical display of RMSE, MAE, and MAPE of four forecasting methods for developing countries.

REFERENCES


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